MATH 347 HW 8

due November 6, at the beginning of class

Homework Guildlines

Obviously, your solutions need to be complete and correct, but to receive full credit your write-up should also satisfy the following:

- All the important logical steps in the proof should be present and fully explained.
- All assumptions should be clearly identified.
- Your solutions should be clear and concise. If a sentence does not further the reader's understanding of the solution then it has no place in your write up.
- Use full and grammatically correct English sentences. Mathematical symbols should be used only to render complex mathematical relationships into a readable form.

Moreover, in order to obtain full credit for the homework, you must write down, in the very least, an attempt at a solution for each problem.

Problems

Do the following problems from your book: 13.31, 13.32, 14.13, 14.15, 14.22. Also answer the following:

(1) Let $\{a_n\}$ be a sequence of real numbers. Let E denote the set of all real numbers $r \in \mathbb{R}$ for which there is a subsequence $\{a_{n_k}\}$ of $\{a_n\}$ which converges to r. Show that E if $\{s_n\}$ is a sequence in E which converges to $s \in \mathbb{R}$, then $s \in E$.[1]

(2) Let E be as in the previous exercise. Define the *upper limit* and *lower limit* of $\{a_n\}$ by

$$\limsup_{n\to\infty} a_n := \sup E \qquad\qquad \liminf_{n\to\infty} a_n := \inf E.$$

Show the following:

(a) Use the previous exercise to show that $\limsup_n a_n$ is actually the limit of a subsequence of $\{a_n\}$.

[1] One says that E is a closed subset of the real line.

(b) Show that if $\limsup_n a_n < x$ then for some N, $a_n < x$ for all $n \geq N$. State and prove the analogous statement for lim inf. (Hint: Use Bolzano-Weierstrass.)

(c) the sequence $\{a_n\}$ converges if and only if $\limsup_n a_n = \liminf_n a_n$, in which case the common value is $\lim_n a_n$. (Hint: Use part (b).)

(d) Calculate the upper and lower limits of the sequence $a_n = (-1)^n(1 + 1/n)$.